Discussion 12/10 (Last One)

Exam review.

Vectors: what they are.

find angles btw vectors, directions of vectors, position vector of a pt., parallel vectors, perpendicular vectors.

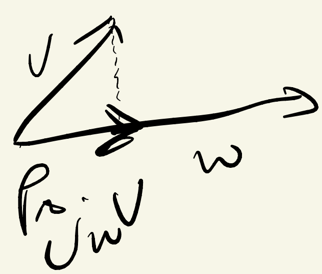
dot prod.

$v \times w = 0$ $\quad$ $v \cdot w = 0 \Rightarrow v \perp w$.

$v \parallel w$.

Geometric interpretation of $\cdot$ and $\times$.

- $\cdot$:



$|\mathrm{Proj}_w v| \cdot |w|$.

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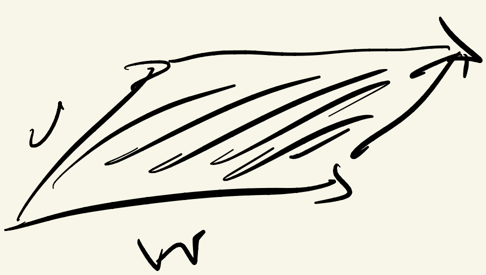


$v \times w = A$.

Equations of lines & planes

Lines: $\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t \boxed{\langle a, b, c \rangle}$.

planes: $\vec{n} \cdot \langle x_0, y_0, z_0 \rangle = 0$.

If lines are parallel.

plane are parallel

planes are perpendicular.

line of intersection of two planes:

$(n_1 \times n_2)$

~~Vector-Valued Function, Parametric Curves.~~

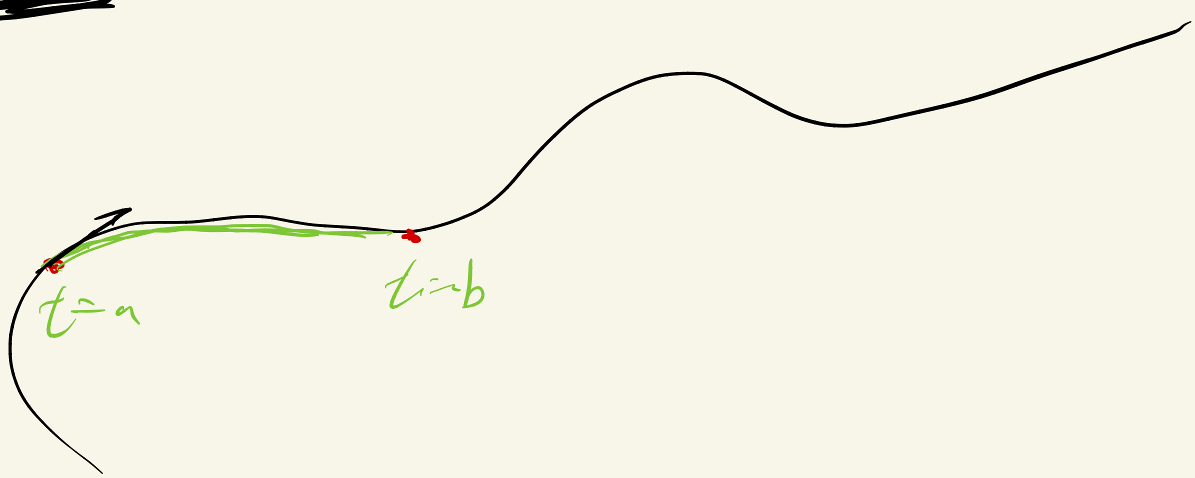
$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle\ t \langle a, b, c \rangle$$

$$= \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

$$a \le t \le b$$

$$\vec{r}'(t) = \left\langle \frac{dx(t)}{dt}, \frac{dy(t)}{dt}, \frac{dz(t)}{dt} \right\rangle$$



arc-length & curvature.

$$a \le t \le b$$

$$L = \int_a^b |r'(t)|\, dt$$

$$k = \boxed{\left| \frac{dT}{dS} \right|}$$

s = arc-length parameter

$\vec{r}(t)$

$$S(t) = \int_0^t |r'(u)|\, du$$

$s = \underline{\quad} t$

solve for t

$\vec{r}(s)$

$$\left| \frac{d \frac{\vec{r}'(s)}{|r'(s)|}}{dS} \right|$$

$$\frac{|T'(t)|}{|r'(t)|}$$

Partial Derivatives

Know how to compute $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$.

$$\frac{\partial}{\partial x} \neq \frac{d}{dx}$$

Tangent plane of a surface.

tangent plane at (x_0, y_0)

$$z - z_0 = \frac{\partial f(x_0,y_0)}{\partial x}(x-x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y-y_0)$$

of

$$\boxed{z=f(x,y)}$$

Linear approx.

Chain Rule.

$z = f(x,y) \qquad x = g(t) \qquad y = h(t)$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Implicit differentiation

Directional Derivatives & Gradient.

$$\nabla f(x,y,z) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$\boxed{D_{\hat{u}} f(x,y,z) = \boxed{\nabla f(x,y,z)} \cdot \hat{u}}$$

$\hat{u} = \nabla f(x,y,z)$

when

f increases most rapidly.

Min/Max.

local min/max, global min/max.

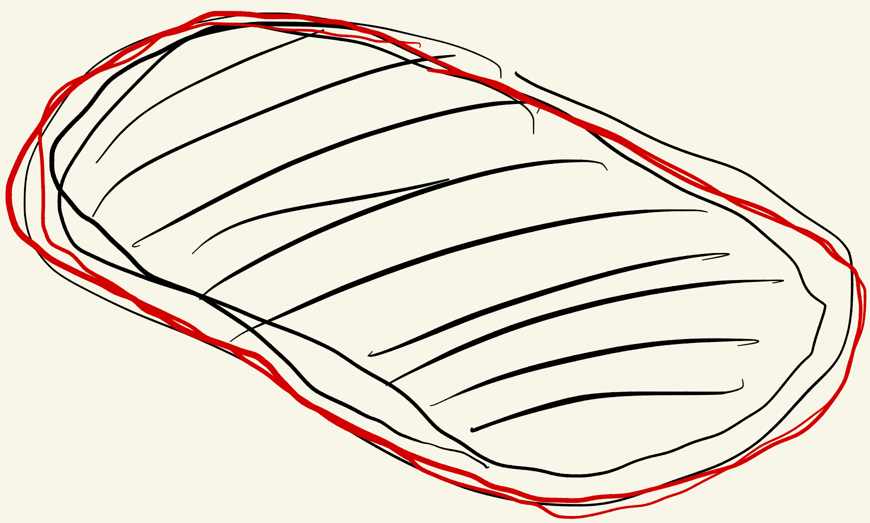
first derivative test: critical points.

vanishing first partials.

Second derivative test: to tell you the "type" of critical point.

EVT: $f(x,y)$ is defined on a closed & bounded set.

$\Rightarrow$ global min/max exist.

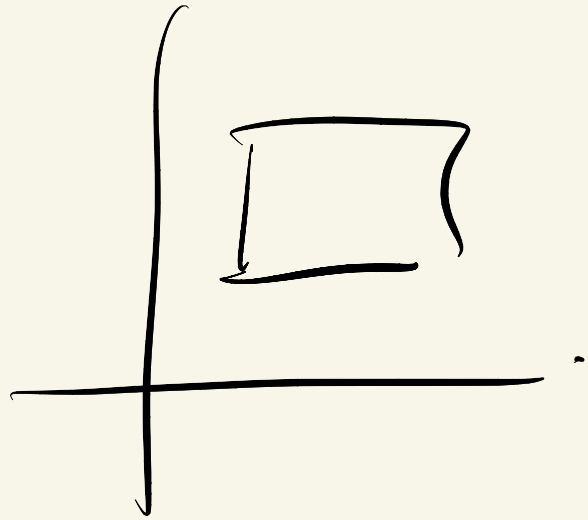


Lagrange Multiplier.

Double integrals

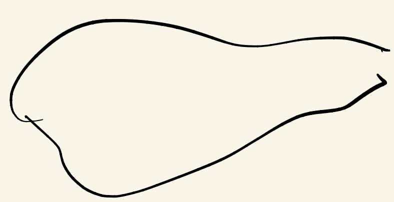
$$\int_a^b \int_c^d f(x,y)\, dx\, dy$$

$$\iint$$



$$\int_c^d \int_a^b f(x,y)\, dx\, dy$$

Over general regions



Type I & Type II.

Polar coordinates

cylindrical

spherical.

Change-of Variables.

$(x, y) \mapsto (u, v)$

Jacobian

$x = h(u, v)$

$y = g(u, v)$

Jacobian.

$$\iint_R f(x,y)\, dA = \iint f(u,v) \left| \quad \right| d$$

Vector Fields

Conservative Vector Field.

$$F = \nabla f$$

$$F = P(x,y)\, i + Q(x,y)\, j$$

$$\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$$

F.

Simply - Connected Region. $C = \vec{r}(t)$

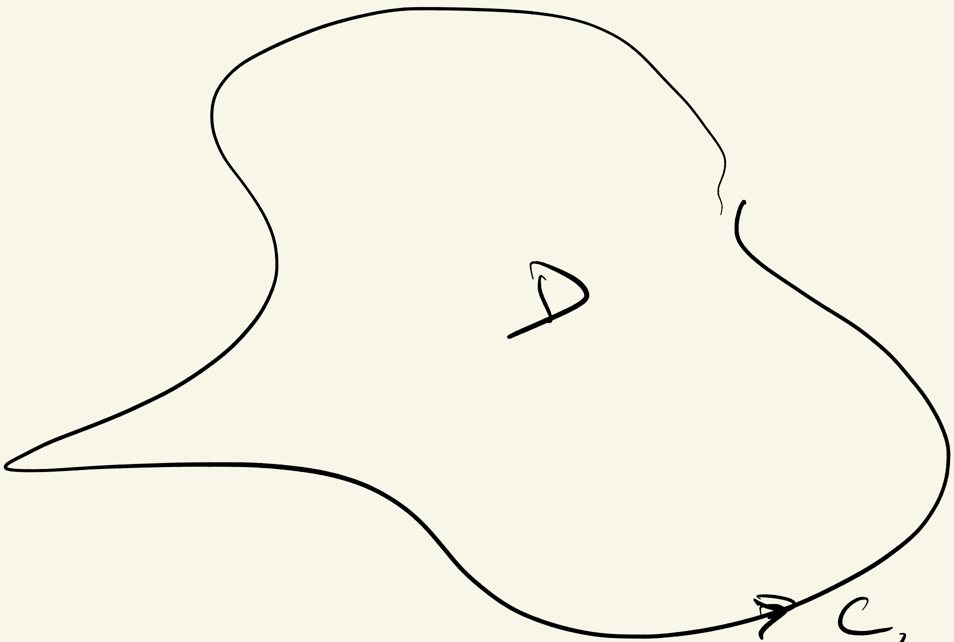
$$\int_a^b F \cdot d\vec{r}$$

If F Conservative

$F = \nabla f$

$$= f(\vec{r}(b)) - f(\vec{r}(a))$$

Green's Thm.



$$\int_C P\,dx + Q\,dy = \int_C P\,dx + \int_C Q\,dy$$

$$= \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$= \int_{C_1} P\,dx + Q\,dy + \int_{C_2} P\,dx + Q\,dy.$$

find f s.t. $Df = F$.

$$F = P(x,y)\,\hat{i} + Q(x,y)\,\hat{j}$$

$$\int P(x,y)\,dx$$

$$\parallel$$

$$\rightarrow \underline{\qquad\qquad} + \underline{g}(y) = f$$

$$\frac{\partial f}{\partial y} = \underbrace{\underline{\qquad\qquad\qquad} + g'(y)}_{= Q(x,y)}$$