Discussion 12/10 2:00 p.m.

Vectors: $\mathbb{R}^2$, $\mathbb{R}^3$

angles b/w vectors, direction of vectors, magnitude of vector, parallel vectors,

perpendicular

$\boxed{v \cdot w = 0}$

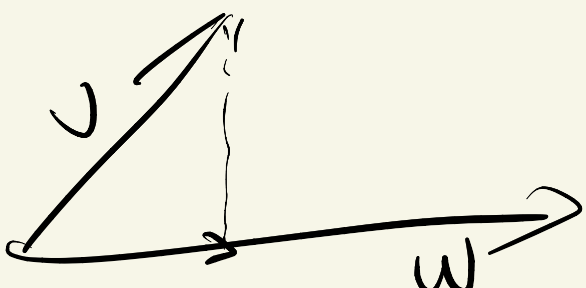
$\Downarrow$

$v \perp w$

$\boxed{v \times w = 0}$

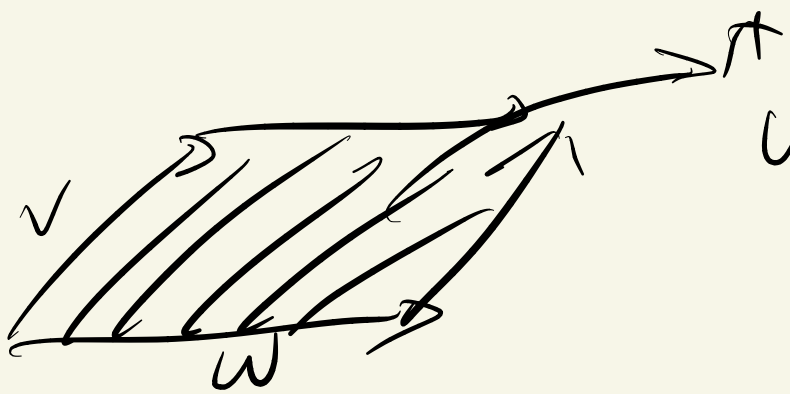
$\Longleftrightarrow$ $v \parallel w$

Geometric interpretation of $\cdot$, $\times$



$\mathrm{Proj}_w v$

$v \cdot w = |\mathrm{Proj}_w v|\,|w|$



$v \times w = A.$

position vector of a point (x, y, z) is $\langle x, y, z \rangle$.

Equations of lines & planes.

$$\vec{r}(t) = \underbrace{\langle x_0, y_0, z_0 \rangle}_{\text{a point on line}} + t \underbrace{\langle a, b, c \rangle}_{\text{direction}}.$$

$$\vec{n} \cdot \langle x_0, y_0, z_0 \rangle = 0.$$

parallel lines, parallel planes, line of intersection of planes.

$\vec{n}_1 \times \vec{n}_2$

Vector valued function, Parametrized Curves

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\boxed{\vec{r}'(t) = \left\langle \frac{dx}{dt}(t), \frac{dy}{dt}(t), \frac{dz}{dt}(t) \right\rangle}$$

arc-length $a \leq t \leq b$.

$$\int_a^b |r'(t)|\, dt.$$

arc-length parametrization.

$$\underline{s(t)} = \int_0^t \underline{|r'(u)|\, du}.$$

$\vec{r}(s)$.

Curvature:

$$T = \frac{r'(t)}{|r'(t)|}$$

$$\kappa = \left| \frac{dT}{ds} \right|$$

$$= \frac{|T'(t)|}{|r'(t)|}$$

Ch. 11.

Partial derivatives

$f(x, y, z)$

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial f}{\partial z}.$$

Chain Rule:

$$z = f(x,y) \qquad x = g(t)$$
$$y = h(t)$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}.$$

Tangent Planes.

$$\boxed{z = f(x,y)} \qquad (x_0, y_0)$$

$$z - z_0 = \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$$

$$\underbrace{x^2+y^2+z^2}_{F(x,y,z)} = 2.$$

$$z = \pm\sqrt{2-x^2-y^2}$$

$\frac{\partial z}{\partial x}$ ...

DF

Directional derivatives, Gradient

$f(x,y,z)$

$Df(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$

$D_{\hat{u}} f = \underline{Df \cdot \hat{u}}$ ∞

$\hat{u} \perp Df$

$$\boxed{D_{\hat{u}} f(x_0, y_0)}$$

Min/Max.

Local min/max

find this by using

1st derivative test

to find

critical pts

Second der. test

Global min/max.

E.V.T. existence where f is defined on a closed & bdd set.

Check boundary

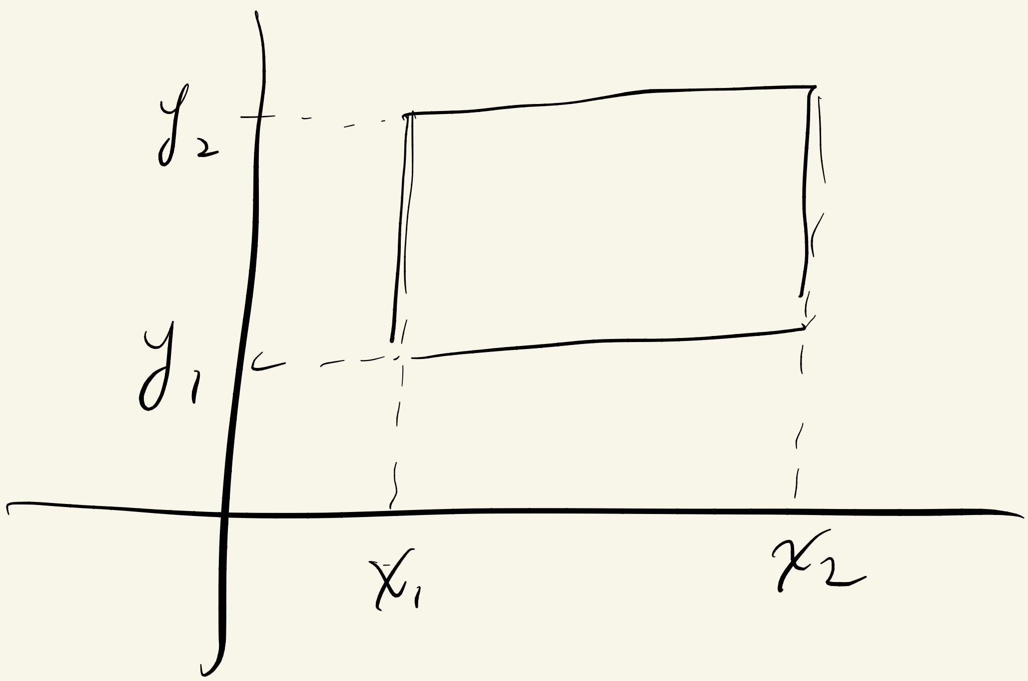
Lagrange Multipliers.

Chapter 12

double integrals:

$$\iint_R f(x,y)\,dA$$

= Volume in 3D, under the graph of f.



$$\int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x,y)\,dy\,dx$$

$$\shortparallel$$

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x,y)\,dx\,dy$$

Type I , Type II region.

$c_1 \le x \le c_1$ $\qquad\qquad$ $c_1 \le y \le c_2$

$h_1(x) \le y \le h_2(x)$ $\qquad\qquad$ . . . .

Polar coordinates.

$x = r\cos\theta$ $\qquad\qquad$ $x^2 + y^2 = r^2$

$y = r\sin\theta$

$$\int\int f(u,v) \cdot r \, dr\, d\theta$$

$$\int\int\int \qquad dx\,dy\,dz$$

Change-of-Variables

definitions.

$$\begin{matrix} x \\ y \end{matrix} \longmapsto \begin{matrix} u \\ v \end{matrix}$$

$$x = f(u,v)$$

$$y = h(u,v).$$

$$\boxed{\text{Jacobian: } \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}}$$

$$\iint_{R} f(x,y)\,dA = \iint_{S} f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$

Chapter 13

Vector fields.

Conservative vector field.

$$F = \nabla f$$

$$F(x,y) = P(x,y)\,\hat{i} + Q(x,y)\,\hat{j}$$

if F is Cons. then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

if ↗ true, and if F is defined on an open simply-connected region,

then

F conservative.

Line integrals. C is $\vec{r}(t)$

$a \le t \le b$

$$\int_C f(x,y)\, ds$$

$$\shortparallel$$

$$\int_a^b f(x(t), y(t))\, |r'(t)|\, dt.$$

$$\int_C f(x,y)\, dx = \int f(x(t), y(t))\, x'(t)\, dt$$

$$\int_C f(x,y)\, dy = \int \cdots\; y'(t)\, dt.$$

$r_0 = (x_0, y_0)$ $\quad r_1 = (x_1, y_1)$

$$\vec{r}(t) = (1-t)\, r_0 + t\, r_1$$

$0 \le t \le 1$ $\quad t=0. \to r_0$

$t=1 \to r_1$

$F \qquad C \quad \vec{r}(t)$

$$\int_C F \cdot d\vec{r} \qquad F = \underbrace{P\hat{i}}\ + \underline{Q}\hat{j}$$

$$= \int F(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$= \boxed{\int P\,dx + Q\,dy}$$

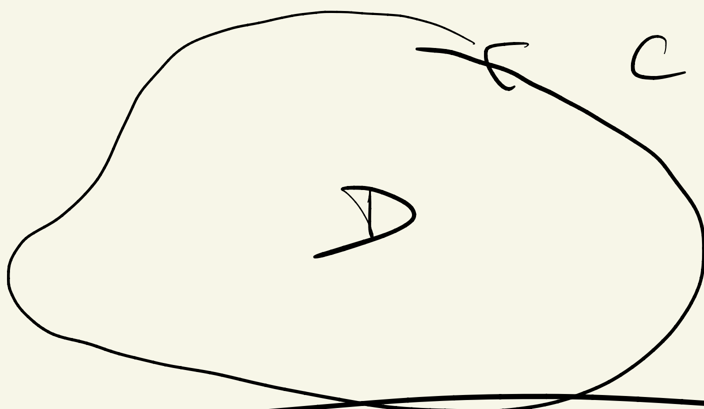
$$= \iint_D (\qquad) \_\_\_$$

Fundamental Thm. of Line Integrals.

If F conservative

$$F = \nabla f$$

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$



$$\boxed{\int P(x,y)\,dx + Q(x,y)\,dy = \iint_D \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) dA.}$$

Extended Version of Green's Thm.

Pg. 938 – 939.

$$F = P(x,y)\,i + Q(x,y)\,j$$

f

$$\int P(x,y)\,dx = \underline{\qquad} + g(y)$$

f

$$\frac{\partial f}{\partial y} = \text{-----} + g'(y)$$

compare with $Q(x,y)$

$$A = \oint_C x\,dy$$

$$= -\oint_C y\,dx$$

$$= \frac{1}{2}\oint_C x\,dy - y\,dx$$