**Math 2202: Multi-variable calculus** **Instructor: Qile Chen**

due 10/16 2:00pm Mock Midterm exam 2 90 minutes

Name: Section:

**Instructions.**

(1) No quiz this week, but there will be a quiz next week after the midterm.

**Question 1** Consider a curve $\vec{r}(t) = (2t, 1-3t, 5+4t)$ for $t \geq 0$.

**(1)** Calculate the length of the curve with $0 \leq t \leq t_0$.

**(2)** Find the arc-length parameterization $\vec{r}(s)$ where $s$ is the arc-length parameter.

**(3)** Calculate $\frac{\mathrm{d}\,\vec{r}}{\mathrm{d}\,s}$ and $\frac{\mathrm{d}^2\,\vec{r}}{\mathrm{d}\,s^2}$.

**(4)** Calculate the curvature $\kappa(s)$ of the curve using the arc-length parameter.

**Question 2** Consider the function $f(x, y, z) = \sin(3x + yz)$ and the unit vector $\vec{u} = (\frac{1}{3}, \frac{2}{3}, 1)$. Find:

(1) The partial derivative of $f$ with respect to $x$.
(2) The partial derivative of $f$ with respect to $y$.
(3) The partial derivative of $f$ with respect to $z$.
(4) The gradient of $f$.
(5) The directional derivative of $f$ along/in the direction of $\vec{u}$.

**Question 3** Consider the function $f(x, y) = \ln(x^2 + 4y^2)$.

(1) What is the domain of definition of $f(x, y)$?
(2) What is the range of the function $f(x, y)$?

**Question 4**

(1) Write down a vector valued function $\vec{r}(t)$ that corresponds to a circle on the plane centered at 0 with radius 1.
(2) Compute the curvature of the circle.

**Question 4** State what it means for a function $f(x, y)$ to be continuous at a point $(a, b) \in \mathbb{R}^2$.

**Question 5 [10 points]** Find the limit, if it exists, or show that the limit does not exist.

**(1)** $\lim_{(x,y)\mapsto(0,0)} \dfrac{\sin(xy)}{xy}$.

**(2)** $\lim_{(x,y)\mapsto(0,0)} \dfrac{y^4}{x^4+3y^4}$.

**Question 6**

**(1)** Find the linear approximation of $f(x,y) = 3y^2 - 2x^2 + x$ at the point $(x,y) = (1,1)$. Simplify your answer to the form $L(x,y) = Ax + By + C$.

**Question 7**

Review implicit differentiation and make sure you know how to do it, specifically be able to use Equation 6 and Equation 7 in Section 11.5.